Efficient Estimation of Pairwise Effective Resistance

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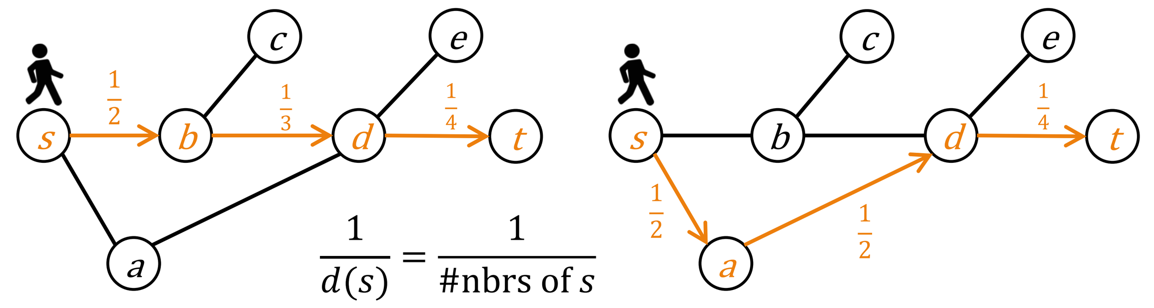
What is Effective Resistance (ER)?

- ***i*-th hop Random Walk Probability**



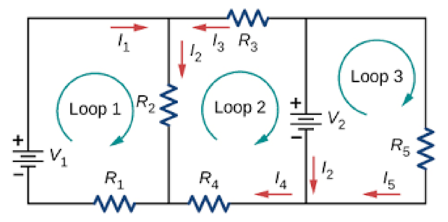
$p^i(s,t)$ = Pr[A random walk starting from s visits t at the i-th hop]

- **Effective Resistance**

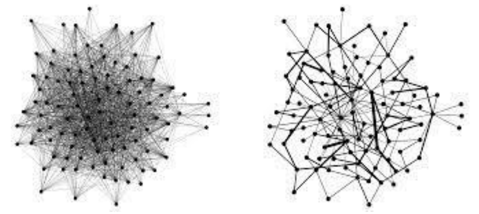
$$ER(s,t) = \sum_{i=0}^{+\infty} \frac{p^i(s,s)}{d(s)} + \frac{p^i(t,t)}{d(t)} - \frac{p^i(s,t)}{d(t)} - \frac{p^i(t,s)}{d(s)}$$

The ER of node pair (s,t) is a sum of the random walk probabilities of all possible numbers of hops between s and t. It describes the *dissimilarity* of nodes s and t.

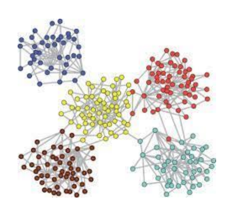
- **Applications of ER**
  - **Electrical Circuit & Power Network Analysis**



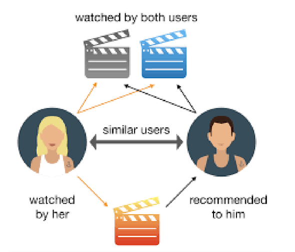
- **Graph Sparsifiers**



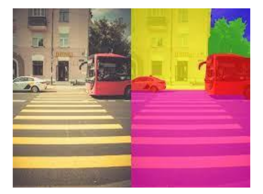
- **Graph Clustering**



- **Recommendation Systems**

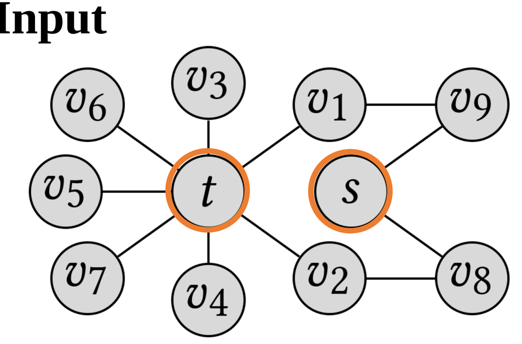


- **Image Segmentation**



Problem Definition

- **ϵ-Approximate Pairwise ER Query**
  - **Input**



An undirected & unweighted graph G

Two nodes s and t

Error threshold ϵ

- **Output**

$$|ER(s,t) - ER'(s,t)| \le \epsilon$$

Exact ER — Approximate ER

Limitations of Existing Work

- **Existing works need**
  - **either expensive matrix operations**
  - **or a huge number of long random walks**
- **The SOTA is a Monte-Carlo Approach**
  - **Maximum Random Walk Length**

$$\ell = \left\lceil \frac{\log\left(\frac{4}{\epsilon(1-\lambda)}\right)}{\log(1/\lambda)} \right\rceil$$ **Too Large!**

λ is the 2nd largest eigenvalue of G's transition matrix, which is usually large (> 0.9) and thus leads to a large ℓ

- **Total #Random-Walks Needed (failure prob.=1%)**

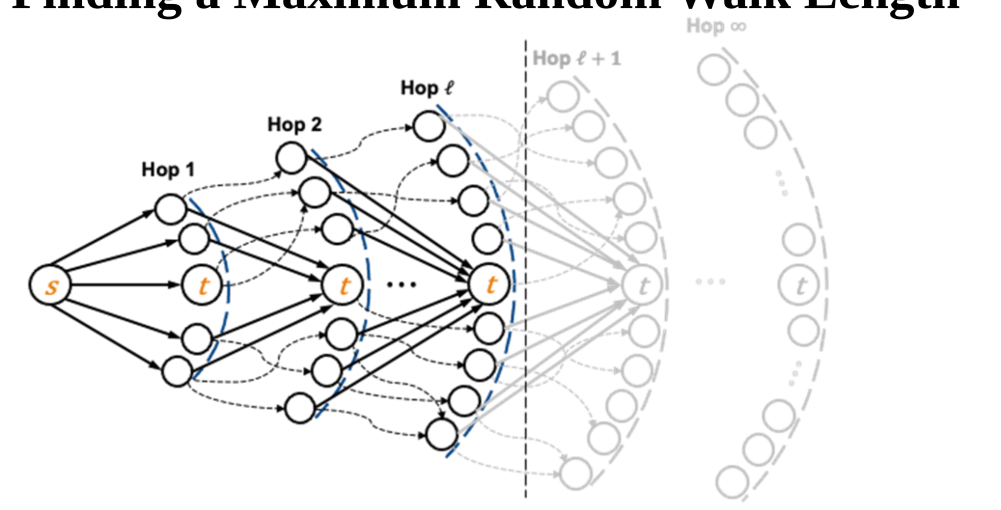
$$r = \frac{40}{\epsilon^2} \cdot \ell^2 \ln(800\ell)$$ **Too Large!**

- **On Facebook graph (#nodes=4k, #edges=88k)**

	$\epsilon = 0.5$	$\epsilon = 0.1$	$\epsilon = 0.05$
ℓ	42	57	64
r	2.94×10^6	139×10^6	710×10^6

Reducing Random Walk Length

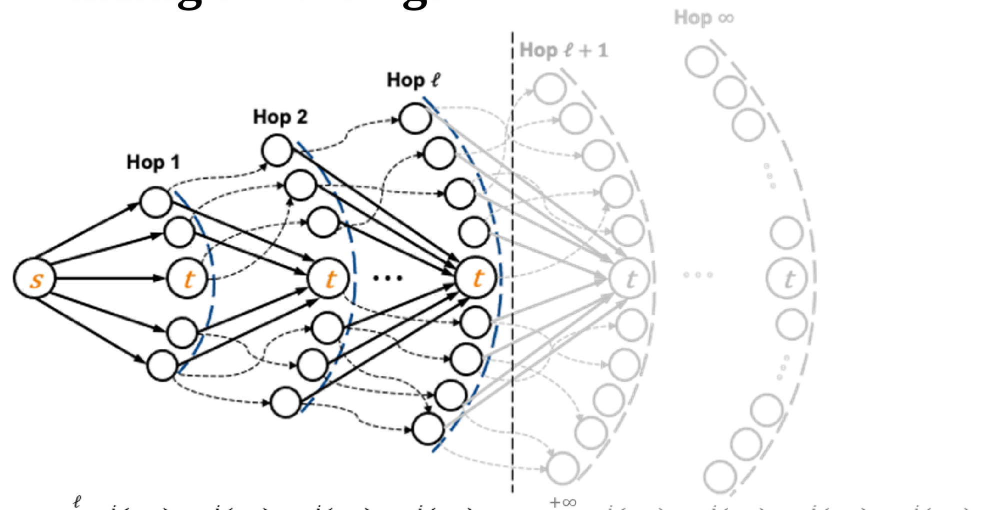
- **Finding a Maximum Random Walk Length**



$$ER(s,t) = \sum_{i=0}^{\ell} \frac{p^i(s,s)}{d(s)} + \frac{p^i(t,t)}{d(t)} - \frac{p^i(s,t)}{d(t)} - \frac{p^i(t,s)}{d(s)} + \sum_{i=\ell+1}^{+\infty} \frac{p^i(s,s)}{d(s)} + \frac{p^i(t,t)}{d(t)} - \frac{p^i(s,t)}{d(t)} - \frac{p^i(t,s)}{d(s)}$$

Estimate this with at most an $\epsilon/2$ error — $\approx \epsilon/2$

- **Utilizing Node Degrees**



$$ER(s,t) = \sum_{i=0}^{\ell} \frac{p^i(s,s)}{d(s)} + \frac{p^i(t,t)}{d(t)} - \frac{p^i(s,t)}{d(t)} - \frac{p^i(t,s)}{d(s)} + \sum_{i=\ell+1}^{+\infty} \frac{p^i(s,s)}{d(s)} + \frac{p^i(t,t)}{d(t)} - \frac{p^i(s,t)}{d(t)} - \frac{p^i(t,s)}{d(s)}$$

Nodes with large degrees have smaller maximum random walk lengths

- **Our Maximum Random Walk Length**

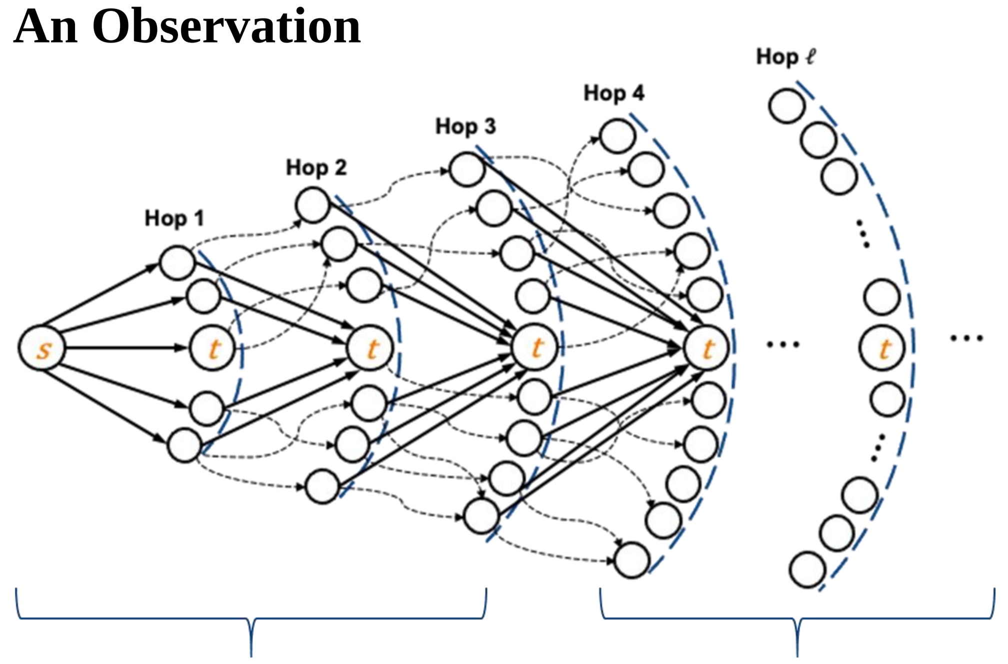
$$\ell = \left\lceil \frac{\log\left(\frac{\frac{2}{d(s)} + \frac{2}{d(t)}}{\epsilon(1-\lambda)}\right)}{\log(1/\lambda)} - 1 \right\rceil$$

- **On Facebook graph (#nodes=4k, #edges=88k)**

	$\epsilon = 0.5$	$\epsilon = 0.1$	$\epsilon = 0.05$
ℓ	42	57	64
Our ℓ	6	21	28
r	2.94×10^6	139×10^6	710×10^6
Our r	48.8×10^3	17.2×10^6	126×10^6

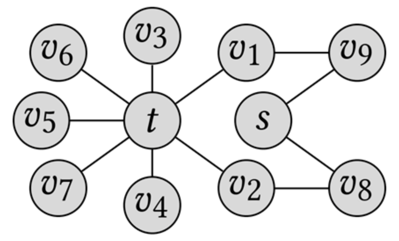
Pruning Random Walks

- **An Observation**



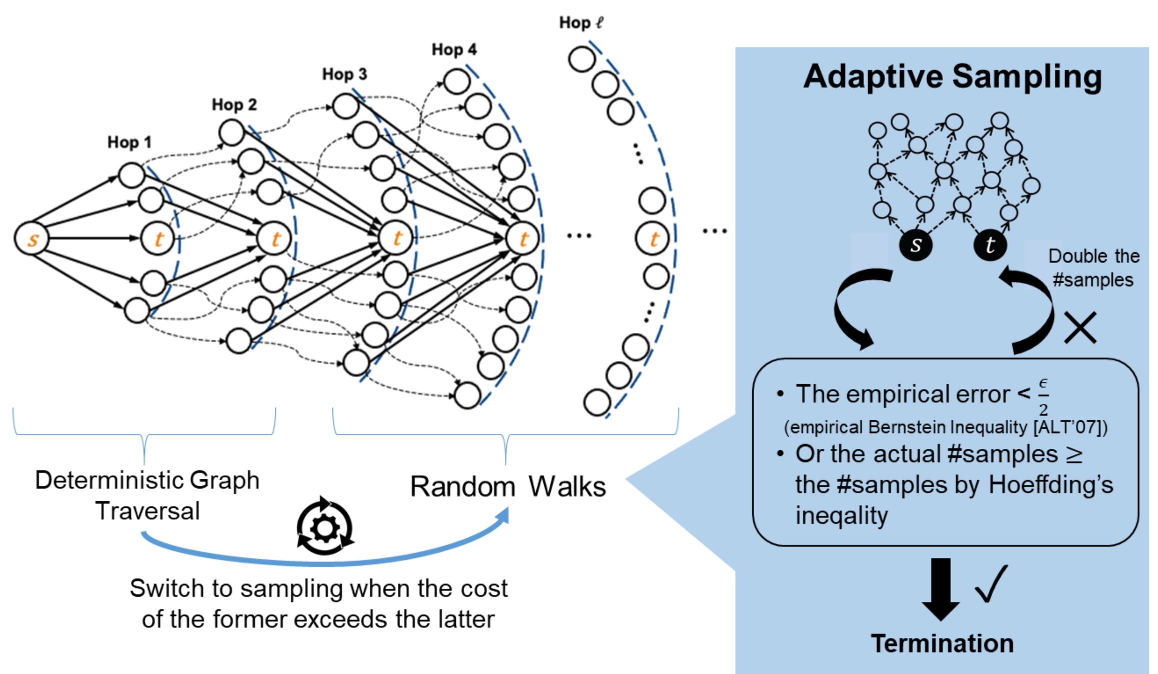
- **#Path & #Random-Walks from Node s to Node t**

Hop	1	2	3	4	5	6	7	8
#paths	9	13	61	97	439	723	**3231**	**5387**
#random walks	**31**	**122**	**275**	**488**	**762**	**1097**	1493	1949



- Deterministic search is better when #hops is small
- Sampling is better when #hops is large

- **Combining Deterministic Graph Traversal and Random Walks in an Adaptive Manner**



Experimental Setup

Dataset	#nodes	#edges	Avg. degree
Facebook	4.04K	88.2K	43.69
DBLP	317.1K	1.05M	6.62
YouTube	1.13M	2.99M	5.27
Orkut	3.07M	117.2M	76.28
LiveJournal	4M	34.68M	17.35
Friendster	65.6M	1.81B	55.06

- Our Methods (AMC (only random walks), GEER (deterministic graph traversal + random walks)) against 5 competitors: EXACT, TP/TPC (KDD'21), RP (STOC'08), and SMM (our baseline)
- Linux machine with an Intel Xeon(R) Gold 6240@2.60GHz 32-core processor and 377GB of RAM

Experimental Results